Handout 12: Moment Generating Functions

The **moment generating function (mgf)** of a random variable X is defined and denoted as:

$$m_X(t) = \mathbb{E}\left[e^{tX}\right], \quad t \in \mathbb{R}.$$

The name of the function comes from the fact that the kth derivative of the mgf, when it exists, yields the kth (raw) moment when evaluated at 0:[1]

$$\frac{d^k}{dt^k} m_X(t)\big|_{t=0} = \mathbb{E}X^k = \mu'_k.$$

The mgf of a sum of independent random variables is equal to the product of the moment generating functions. So, for example, if X and Y are independent and $Z = X + Y$, then:

$$M_Z(t) = M_X(t) \times M_Y(t).$$

This property makes it easy to find the mgf for many classes of distributions that would be difficult to compute directly.

The moment generating function has another property that we will further investigate on a future handout.

[1] A derivation of this comes once again from the Taylor series of e^x around zero, $\sum_i x^n/n!$, that used in the Poisson distribution.