# Worksheet 01 (Solutions)

**1**. We will give more formal definitions later, but for now define a probability of an event to be a number between 0 and 1 that indicates how likely an event would be to happen. For example, a value of 0 indicates that it will never happen, a value of 1 that it will always happen. This matches the way that the word 'probability' is colloquial used in a non-technical context. While in casual conversatoin most people refer to the number as a percentage or fraction, it will be good to start thinking of them as decimals. Given this, give approximate values for the probability of the following events:

(a) A randomly selected M&M will be blue.
(b) A randomly selected car in Virginia is electric.
(c) A randomly selected book starts with the word 'The'.
(d) An NBA basketball player will make a free throw.
(e) A pregnancy results in having twins.
(f) A clover will be a four-leaf clover.
(g) A letter will be lost by the U.S. postal service.
(h) Someone born in the U.S. in the year 2000 is named Taylor.

*Solution:* [Any reasonable answers are fine. I've included values for some of the answers that I found online, but I would not say that these are definitive.]

(a) **A randomly selected M&M will be blue.** 0.189
(b) **A randomly selected car in Virginia is electric.** 0.0027
(c) **A randomly selected book starts with the word 'The'.** ?
(d) **An NBA basketball player will make a free throw.** 0.7–0.8
(e) **A pregnancy results in having twins.** 0.0001
(f) **A clover will be a four-leaf clover.** 0.0001
(g) **A letter will be lost by the U.S. postal service.** 0.03
(h) **Someone born in the U.S. in the year 2000 is named Taylor.** 0.0003

**2**. Many probability theory questions are described in terms of flipping a coin, with the idea that every coin flip results in the coin landing one of the two sides, which we call 'heads' (H) or 'tails' (T). A sequence of coin flips can be written as a sequence of H's and T's. Write down all possible sequences from flipping a coin twice.

*Solution:* HH, HT, TH, TT

**3**. Another common device in probability theory are dice (note that the singular is called a 'die'). The most common type of die are six-sided, but theoretically they can have any number of sides. We can describe a sequence of die flips as a sequence of numbers. What would be equivalent to a 2-sided die?

*Solution:* A two-sided die is just a coin flip.

**4**. Finally, another common device in probability theory is a deck of cards. In this class we will consider a simplified but generalized version of a standard card deck. Each card in our decks will have a suit/color and a number; there will be $C$ suits, with one card of each suit for every integer from 1 to $N$. What would be the values of $C$ and $N$ that reproduce the standard 52-card deck of poker cards?

*Solution:* We would need $C = 4$ (for $\heartsuit$, $\diamondsuit$, $\clubsuit$, $\spadesuit$) and $N = 13$ (2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A).

**5**. Write the sum of the square integers from 1 to N using a summation symbol.

*Solution:*

$$\sum_{i=1}^{N} i^2$$

**6**. Find the derivative of $x^2 e^x$.

*Solution:* Using the chain rule and simplifying.

$$\begin{aligned}\frac{d}{dx}\left[x^2 e^x\right] &= x^2 \cdot \frac{d}{dx}\left[e^x\right] + e^x \cdot \frac{d}{dx}\left[x^2\right] \\ &= x^2 \cdot e^x + 2e^x \cdot x \\ &= x \cdot e^x \cdot (x+2)\end{aligned}$$

**7**. Find the definite integral of $xe^{x^2}$ from 0 to 1.

*Solution:* Using U-substitution with $u = x^2$ and $\frac{1}{2}du = xdx$.

$$\begin{aligned}\int_0^1 \left[xe^{x^2}dx\right] &= \frac{1}{2}\int_0^1 [e^u du]\\ &= \frac{1}{2}\left[e^1 - e^0\right]\\ &= \frac{1}{2}[e-1] \approx 0.859\end{aligned}$$

**8**. What is the value of $log_2(16)$? Do not use a calculator.

*Solution:* $log_2(16) = log_2(2^4) = 4$