Worksheet 17 (Solutions)

1. You want to create a model to show the amount of time in minutes between the arrival of each customer to arrive at your company. A dataset with 10 data points shows that the average arrival rate is one customer every 1.83 minutes.[1] A common model to model arrival times, as we have already mentioned, is an exponential distribution. If A is a random variable with an exponential distribution that models the time between customers, what value of λ would you use? Hint: You want the mean of the distribution to match the mean of the sample.

[1] The actual data are: 0, 0, 1, 1, 1, 2, 2, 2, 2, 3, 3, 5.

Solution: The value of λ is just the inverse of the mean, so $\lambda = \frac{1}{1.83} \approx 0.546$.

2. You want to create a model to show the amount of time in minutes that it takes an employee at your company to complete a transaction with a customer. From a sample of 12 data points, you have observed that the average transaction time is 15.42 minutes and the variance of the transaction time is 32.26 minutes.[2] The Gamma distribution is commonly used to model quantities of this type. (a) If T is a random variable with a Gamma distribution that models the transaction time of a randomly selected customer, what values of α and β would you use? Hint: You want the mean and variance of the sample to match the mean and variance of the data. (b) Given the distribution, write the probability that a transaction is longer than 30 minutes as an integral. You will not be able to easily simplify the result.

[2] The actual data are: 6, 9, 12, 13, 13, 13, 15, 18, 18, 19, 23, 26.

Solution: (a) The expected value is $\alpha\beta$ and the variance is $\alpha\beta^2$. So, the variance divided by the expected value is the value of β:

$$\beta = \frac{32.26}{15.42} = 2.092$$

Then, α is equal to the expected value divided by β:

$$\alpha = \frac{15.42}{2.092} = 7.371$$

The value of (b) is just the integeral of the density from 30 to infinity:

$$\int_{30}^{\infty} \frac{1}{\Gamma(\alpha)\beta^{\alpha}} \cdot x^{\alpha-1} e^{-x/\beta} = \int_{30}^{\infty} \frac{1}{\Gamma(7.371)2.092^{7}.371} \cdot x^{7.371-1} e^{-x/2.092}$$

And that is the form you can leave the answer in.

3. ($\star$) Imagine a store that has customers arriving at rate with the time between customers being modeling by A and the transaction time being modeling by T, as you developed above. You have a total of E

employees that can each work with a single customer at once. When customers arrive at the business, they go to any free employee for help. If all employees are busy, they wait in line until an employee is free. (a) What minimum number of employees do you think are needed so that the length of the line does not tend to grow infinitely long? (b) What do you think is the longest the line will get if we run the simulation for 500 hours with this minimum number? Note: This is a conceptual question, not one that you should be able to have a perfect answer to. However, you should be able to make an intelligent guess for (a). I will show a simulation during class that shows the results.

Solution: (a) If people arrive on average about one every 1.8 minutes and each client takes a little less than 16 minutes to see, a reasonable guess is that we need on average about $16/1.8 \approx 8.8 < 9$ employees. (b) I am not sure how you would approach this; it is more of a fun guessing game than anything else, though I look forward to any interesting justifications!