Worksheet 05

1. Consider ordered sequences of the numbers 0 and 1. For example, 0011011 would be a sequence of length 7. How many unique sequences are there of length n?

2. We flip a fair coin 10 times. What is the probability that exactly 4 of the flips are heads?

3. We flip a fair coin 10 times. What is the probability that there are three or fewer heads.

4. Based on the solutions to the previous three questions, why must the following identity hold?

$$\sum_{j=0}^{n} \binom{n}{j} = 2^n.$$

5. We have a jar with m white marbles and m black marbles. You start by selecting one marble. In addition to putting it back into the jar, you add another marble of the same color. What is the probability that a second marble selected from the jar will have the same color as the first?

6. How many ways are there to permute the letters in the word RAINBOW?

7. How many ways are there to permute the letters in the word STATISTICS?

8. There are m faculty members in the mathematics department and n faculty members in the data science department. We need to form a committee of $k \leq m + n$ faculty members.[1] (a) How many possible committees are there? (b) How many possible committees are there if we know that there are exactly j mathematics faculty on the committee, where j is an integer between 0 and k (inclusive)? (c) Put the two parts together to state and prove and identity for binomial coefficents.

[1] Answer all of the questions in terms of binomial coefficents. Don't try to unpack them.

9. We counted the probability of getting a set of cards that are all the same color/suit by treating different orderings of the cards as distinct. This is fine if we do it consistently in the numerator and denominator. With our new results, we should see that it is easier to do if we count sampling unordered sets. Take a desk of cards with 7 suits and 20 cards of each suit. What is the probability that a hand of 5 cards all have the same suit?