Worksheet 10

1. Let X be a random variable with the following probability mass function:

$$p_X(x) = \begin{cases} 0.6, & \text{if } x = 1 \\ 0.2, & \text{if } x = 2 \\ 0.2, & \text{if } x = 3 \end{cases}$$

Find $\mathbb{E}X$.

2. Consider flipping a four-sided die with sides 1-4. Let X be the outcome of rolling the die once, Y be the independent outcome of rolling the die a second time, and Z be the sum $X + Y$. (a) What are $\mathbb{E}X$, $\mathbb{E}Y$, and $\mathbb{E}Z$? (b) What is the pmf of X? (c) Sketch the cdf of X.

3. Let Y be a random variable with the following probability mass function:[1]

$$p_Y(y) = \begin{cases} (1-a), & \text{if } y = 0 \\ a, & \text{if } y = 1 \end{cases}$$

For some $a \in [0, 1]$. Find (a) $\mathbb{E}Y$ and (b) sketch the cdf $F_Y(y)$.

[1] I change the variable names here to make this more clear.

4. Consider a sequence of N independent tosses of a fair coin. Let $X_1, \dots, X_n$ be a sequence of independent random variables such that X_i is 1 if the ith flip is heads and 0 if it comes up tails. Define $Z = \sum_{i=1}^{n} X_i$, which measures the total number of heads. (a) What is $\mathbb{E}Z$ (Hint: there is an easy way to do this)? (b) What is the pmf of Z? (Hint: we have already done this!).

5. Consider flipping a fair coin until it comes up heads. Let X be a random variable equal to the number of flips that are made. Calculate (a) $p_X(1)$, (b) $p_X(2)$, and (c) $p_X(3)$. (d) Write a general formula for $p_X(n)$. (e) Write down the quantity $\mathbb{E}X$. Notice that the summation is difficult to simplify (you may leave it as is).

6. Let X be a random variable defined as follows:

$$p_X(x) = \frac{1}{n}, \quad x \in \{1, 2, \dots, n\}$$

Calculate $\mathbb{E}X$ and simplify the result.