Exam 03: Takehome Portion

Name: ______________________________

(40 points) This take-home exam is due at the start of class on Monday, 15 April. You must work on your own, but may use any static external references that you need. There is no inclass exam, but you must come on Monday for the project workshop.

1. Let $X \sim Exp(\lambda)$. Compute the Fisher information $\mathcal{I}(\lambda)$.

2. When $n = 1$, the MLE for the parameter λ from the exponential distribution is $\frac{1}{X}$. We cannot compute the variance of this exactly, but asymptotically it will be close to $\frac{1}{Var(X)}$. Using this, what is the asymptotic effiency of the MLE when $n = 1$?

3. Write down the shape (that is, up to a constant) of Jeffreys Prior for the exponential distribution. Like the Poisson, this does not correspond to a known distribution.

4. Using the following link,[1] you can collect data about the weather in Henrico for any recent month:

[1] Use the digital version of the course website to avoid having to write this out.

`https://www.wunderground.com/history/monthly/us/va/henrico/KRIC/date/2023-10`

Going through the data from October 1st through April 1st, record the distances in days between days in which there was at least 1cm (0.4 inches) of rain. Write these numbers below. Assume (incorrectly) that there was rain on September 30th.[2]

[2] If it rained multiple days in a row, those values should be 1 (for one day between them), not zero.

5. If we model the data you collected above as an Exponential distribution, what are the (a) MLE estimator of λ, (b) the Bayesian estimator with prior $Gamma(2, 3)$, and (c) the Bayesian estimator with prior $Gamma(1/2, 8)$?

6. Let Y be the total number of observations you have in question 4. We can model this as $Bin(183, p)$, where p is the probability that it rains on a randomly selected day in Henrico, VA. Using your data, what are (a) the MLE estimator of p, (b) the Bayesian estimator with a prior equal to $Beta(1, 4)$, and (c) the Bayesian estimator using the Jeffreys prior?