Table of Distributions

The following table provides the notation, support, density, mean, variance, moment generating function and the R function to produce a random sample for all of the main distributions we will need. Those in the first block are discrete distributions and the second set are continuous distributions. The first argument of each R function is the number of samples to draw. There are also (using the normal as an example) the following variants with the same parameters: `dnorm` (pmf/pdf), `pnorm` (cdf), and `qnorm` (inverse cdf). The support of the parameters are as follows: $p \in [0,1]$; $k, d_1, d_2 \in \{1,2,\ldots\}$; $\alpha, \beta, \gamma, \lambda, \sigma^2 \in (0,\infty)$; and $\mu \in \mathbb{R}$.

Dist.	**Notation**	**Support**	**PMF/PDF**	**Mean**	**Variance**	**MGF**	**R Code**
Bernoulli	$Bernoulli(p)$	$x \in \{0,1\}$	$p^x(1-p)^{1-x}$	p	$p(1-p)$	$(1-p+pe^t)$	`rbinom(, 1, prob = p)`
Binomial	$Bin(n,p)$	$x \in \{0,\ldots,n\}$	$\binom{n}{x}p^x(1-p)^{n-x}$	np	$np(1-p)$	$(1-p+pe^t)^n$	`rbinom(, size = n, prob = p)`
Geometric	$Geom(p)$	$x \in \{1,2,\ldots\}$	$(1-p)^{x-1}p$	$1/p$	$(1-p)/p^2$	$\frac{pe^t}{1-(1-p)e^t}$	`rgeom(, prob = p) - 1`
N. Binomial	$NB(k,p)$	$x \in \{k,k+1,\ldots\}$	$\binom{x-1}{k-1}(1-p)^{x-k}p^k$	k/p	$k(1-p)/p^2$	$\left(\frac{pe^t}{1-(1-p)e^t}\right)^k$	`rnbinom(, size = n, prob = p) - n`
Poisson	$Poisson(\lambda)$	$x \in \{0,1,\ldots\}$	$\frac{\lambda^x e^{-\lambda}}{x!}$	λ	λ	$e^{\lambda(e^t-1)}$	`rpois(, lambda = lambda)`
Exponential	$Exp(\lambda)$	$x \in [0,\infty)$	$\lambda e^{-\lambda x}$	λ^{-1}	λ^{-2}	$\frac{\lambda}{\lambda-t}$	`rexp(, rate = 1/lambda)`
Gamma	$Gamma(\alpha,\beta)$	$x \in [0,\infty)$	$\frac{1}{\Gamma(\alpha)\beta^\alpha}\cdot x^{\alpha-1}e^{-x/\beta}$	$\alpha\beta$	$\alpha\beta^2$	$(1-\beta t)^{-\alpha}$	`rgamma(, shape = a, scale = b)`
Beta	$Beta(\alpha,\beta)$	$x \in [0,1]$	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\cdot x^{\alpha-1}(1-x)^{\beta-1}$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	$\cdot$	`rbeta(, shape1 = a, shape2 = b)`
Uniform	$U(a,b)$	$x \in [a,b]$	$(b-a)^{-1}$	$\frac{1}{2}(a+b)$	$\frac{1}{12}(b-a)^2$	$\cdot$	`runif(, min = a, max = b)`
Normal	$N(\mu,\sigma^2)$	$x \in \mathbb{R}$	$\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$	μ	σ^2	$\exp\{\mu t+\frac{1}{2}\sigma^2t^2\}$	`rnorm(, mean = mu, sd = sqrt(s2))`
Chi-squared	$\chi^2(k)$	$x \in [0,\infty)$	$\frac{1}{2^{k/2}\Gamma(k/2)}\ x^{k/2-1}e^{-x/2}$	k	$2k$	$(1-2t)^{-k/2}$	`rchisq(, df = k)`
Student-T	$t(k)$	$x \in \mathbb{R}$	$\cdot$	0	k / (k - 2)	$\cdot$	`rt(, df = k)`
F-Dist.	$F(d_1,d_2)$	$x \in [0,\infty)$	$\cdot$	$\frac{d_2}{d_2-1}$	$\cdot$	$\cdot$	`rf(, df1 = d1, df2 = d2)`
Cauchy	$C(\gamma)$	$x \in [0,\infty)$	$\frac{1}{\pi/\gamma(1+(x/\gamma)^2)}$	und.	und.	und.	`rcauchy(, scale = gamma)`

DERVIED DISTRIBUTIONS

- If $X \sim N(\mu,\sigma^2)$ then $aX+b \sim N(a\mu+b, a^2\sigma^2)$ for $a \neq 0$.
- If $X_1, \ldots X_n$ are independent normals, then $\sum_i X_i \sim N(\cdot,\cdot)$.
- A sum of i.i.d. r.v.s with finite variances will be approximately normal (CLT).
- If $Z_1, \ldots Z_n \overset{iid}{\sim} N(0,1)$ then $\sum_i Z_i^2 \sim \chi^2(n)$.
- If $X \sim \chi^2(k_1)$ and $Y \sim \chi^2(k_2)$ are independent, then $X+Y \sim \chi^2(k_1+k_2)$.
- If $Z \sim N(0,1)$ and $C \sim \chi_2(k)$ are independent, then $Z/\sqrt{C/k} \sim t(k)$.
- If $T \sim t(k)$ then T will be approximately $N(0,1)$ for large k.
- If $C_1 \sim \chi^2(k_1)$ and $C_2 \sim \chi^2(k_2)$ are independent, then $\frac{C_1/k_1}{C_2/k_2} \sim F(k_1,k_2)$.