Handout 08: Multiple Means

Today we build an hypothesis test to test the null hypothesis that the means of multiple groups (more than two) are all the same. The alternative hypothesis is simply that at least one of the means is different from the others. The key difficulty of this task is notational.

Consider a set of N independent random variables that are split into K pre-defined **blocks** of sizes $n_1, \ldots, n_k$. We have the following assumption about the distribution of block j:

$$X_{1,j}, \ldots, X_{n_j,j} \overset{iid}{\sim} N(\mu_j, \sigma^2)$$

Notice that the variances are all assumed to be the same and we assume from the outset that the variables have normal distributions. The summary statistics $\bar{X}_j$ and S_j^2 correspond to the sample mean and sample variance of the jth block by itself. The value $\bar{X}$ is the sample average across all observations.

As mentioned above goal is to produce a pivot for the hypothesis:

$$H_0 : \mu_1 = \cdots = \mu_j$$
$$H_A : \exists j \exists k (\mu_j \neq \mu_k)$$

The pivot will have an F distribution. We will derive the form on today's worksheet. Note that unlike the previous tests we have derived, this one does not have a direct corresponding confidence interval because they hypothesis is not a simple, univariate statement. The resulting test is called a **one-way analysis of variance** or **one-way ANOVA**.