Worksheet 06 (Solutions)

1. Consider a random sample $X_1, \ldots, X_n \sim N(\mu_X, \sigma_X^2)$. Write a test statistic to test the null hypothesis that $H_0 : \mu_X = 4$. Write down the rejection region for a confidence level $(1 - \alpha)$.

Solution: The test statistic will be:

$$T = \frac{\bar{X} - 4}{\sqrt{S_X^2 / n}}$$

It will have a T-distribution with $n - 1$ degrees of freedom. This gives a rejection region of:

$$R = \{|T| > t_{\alpha/2}\}$$

2. Consider a two-sample design with the notation in Handout 5, with the assumption of normality and equal variances. Write a test statistic to test the null hypothesis that $H_0 : \mu_X - \mu_Y = 0$. Write down the rejection region for a confidence level $(1 - \alpha)$.

Solution: The test statistic will be:

$$T = \frac{\bar{X} - \bar{Y}}{\sqrt{S_p^2 \cdot \left[\frac{1}{n} + \frac{1}{m}\right]}}$$

It will have a T-distribution with $n + m - 2$ degrees of freedom. This gives a rejection region of:

$$R = \{|T| > t_{\alpha/2}\}$$

3. Consider a two-sample design with the notation in Handout 5, with the assumption of normality. Write a test statistic to test the null hypothesis that $H_0 : \sigma_X^2 = \sigma_Y^2$. Write down the rejection region for a confidence level $(1 - \alpha)$.

Solution: The test statistic will be (you can transpose the σ_Y^2 and σ_X^2 as long as you swap the degrees of freedom as well).

$$F = \frac{\sigma_Y^2}{\sigma_X^2} \sim F(n-1, m-1)$$

The rejection region will be:

$$R = \{F < f_{1-alpha/2}\} \cup \{F > f_{\alpha/2}\}.$$