Worksheet 07 (Solutions)

1. Consider a sample of size 5 with the following values: 0, 1, 5, 7, 12. What are the sample mean and sample variance $\bar{X}$ and S_X^2?

Solution: The sample mean is:

$$\bar{X} = \frac{0+1+5+7+12}{5} = 5$$

And the sample variance is:

$$S_X^2 = \frac{(0-5)^2 + (1-5)^2 + (5-5)^2 + (7-5)^2 + (12-5)^2}{5-1} = 23.5$$

2. Consider collecting data from two populations. We collect $n = 25$ observations from the first group, with sample mean 7 and sample variance 9. From the second group, we have $m = 30$ samples and a mean of 4 with a sample variance of 4. What is the pooled variance S_p^2?

Solution: The pooled variance is just the weighted averages of the sample variances. That is:

$$\begin{aligned} S_p^2 &= \frac{(n-1)S_X^2 + (m-1)S_Y^2}{n+m-2} \\ &= \frac{(24)\cdot 9 + (29)\cdot 4}{53} = 6.264 \end{aligned}$$

3. Using the data from above, construct a 99% confidence interval for the difference in the means. You can use the fact that $t_{0.01/2}(53) = 2.671$.

Solution: Using the formula from the table, we have:

$$\begin{aligned} (\bar{X} - \bar{Y}) \pm t_{\alpha/2} \cdot \sqrt{S_p^2 \times \left[\frac{1}{n} + \frac{1}{m}\right]} \\ (7-4) \pm 2.671 \cdot \sqrt{6.264 \times \left[\frac{1}{25} + \frac{1}{30}\right]} \\ (3) \pm 1.8103 \end{aligned}$$

Which we could also write as: $[1.19, 4.81]$.

4. Using the data from above, run a hypothesis test to see if the samples come from distributions with the same variance using a 99% confidence level. Use the fact that $f_{0.01/2}(24, 29) = 2.76$ and $f_{1-0.01/2}(24, 29) = 0.347$.

Solution: The test statistic for the ratio of the variances is $\frac{S_X^2}{S_Y^2} \cdot \Delta_0$ where Δ_0 is the ratio of the variances. Here $\Delta_0 = 0$, so the test statistic is $F = \frac{9}{4} = 2.25$. The rejection region is:

$$\{F \leq f_{0.01/2}\} \cup \{F \leq f_{0.01/2}\}$$
$$\{F \leq 0.347\} \cup \{F \leq 2.76\}$$

Our F is equal to 2.25, so neither in the left part nor the right part of the rejection region. Therefore, we **fail to reject the null hypothesis**.