Worksheet 05

1. We will start with the easier, though much less commonly used, task of comparing the variances between the samples before moving onto the confidence interval of the mean. Using what we know about the distributions of S_X^2 and S_Y^2, build a pivot based on the scaled ratio of these two quantities that has an F-distribution.

2. Rearrange your previous result to get a confidence interval for the ratio σ_Y^2/σ_X^2. Note that the F distribution is not symmetric.

3. What is $\mathbb{E}[\bar{X} - \bar{Y}]$? Make use of the properties that we already know to make this relatively easy. You should see that this is an unbiased estimator of the difference in the means.

4. What is $\text{Var}[\bar{X} - \bar{Y}]$? Make use of the properties that we already know to make this relatively easy. The result should imply that the difference is a consistent estimator of the difference in sample means.

5. If $\mathcal{G}_X$ and $\mathcal{G}_Y$ are both normally distributed, then $\bar{X} - \bar{Y}$ also has a normal distribution. As we did in the one-sample case, construct a pivot Z that scales this difference to have a standard normal distribution.

6. Assume that $\sigma_X^2 = \sigma_Y^2$, which we can write as just σ^2. Take the definition of the pooled sample variance and multiply both sides by $(n + m - 2)$ and divide by σ^2. If we assume that $\mathcal{G}_X$ and $\mathcal{G}_Y$ are both normally distributed, show that S_p^2 is a scaled version of a chi-squared distribution. What are its degrees of freedom?

7. Put together the previous results to generate a pivot T that has a T-distribution. Rearrange the terms to get a confidence interval for the difference in means.