

## Worksheet 17

1. Consider a prior distribution  $p \sim \text{Beta}(\alpha, \beta)$  for some fixed  $\alpha$  and  $\beta$  for a likelihood given by  $X|p \sim \text{Bin}(n, p)$ . Derive the posterior distribution  $p|X$ .

2. For reasons that we will explore in more next time, the Bayesian point estimator (the best single-number estimator) is the expected value of the posterior distribution. Under the set up from the previous question, what is the Bayesian point estimator  $\hat{p}$  in terms of  $X$ ,  $\alpha$ , and  $\beta$ ?

3. Consider observing  $X \sim \text{Bin}(n, p)$ . We know that the MLE estimator of  $p$  is given by  $\hat{p}_{MLE} = X/n$ . The Binomial comes from doing  $n$  Bernoulli trials and adding the number of 1s. Consider creating a new  $Y$  in which we artificially augment the data  $X$  by adding (in effect) an extra 0 and an extra 1. In other words, we create a  $Y = X + 1$  with the assumption that  $Y \sim \text{Bin}(n + 2, p)$ . What is the MLE of  $p$  using the data from the augmented data  $Y$ ? Where have you seen this before?

4. Consider your solution to the previous set of questions. If  $\alpha$  and  $\beta$  are non-negative integers, how could you describe the Bayesian estimator based on adding data to  $X$ ?

5. The standard uniform distribution is equivalent to  $\text{Beta}(1, 1)$ . In the notes I started by implicitly assuming that this was a fairly neutral starting assumption for indicating that we do not have any strong prior knowledge of  $p$ . Based on your results above, what would actually seem to be the best neutral position if we do not want the prior to have a strong influence on the posterior mean?